Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 1 (Group)

香港数学竞赛 **(1998 – 99)**

决赛项目 **1 (**团体**)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 设 $x \vartriangle y = x + y - xy$，其中 x、y 为实数。若 $a = 1 \vartriangle (0 \vartriangle 1)$，求 a 之值。

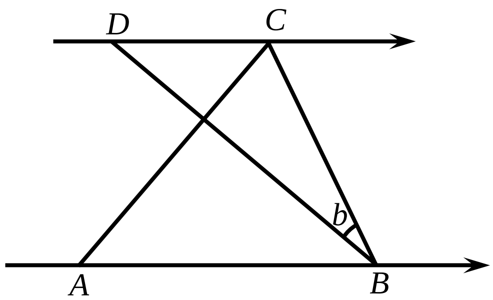
Let $x \vartriangle y = x + y - xy$, where x, y are real numbers. If $a = 1 \vartriangle (0 \vartriangle 1)$, find the value of a.

$a =$

(ii) 在图一，AB 平行 DC，$\angle ACB$ 为一直角，$AC = CB$ 及 $AB = BD$。若 $\angle CBD = b$，求 b 之值。

In Figure 1, AB is parallel to DC, $\angle ACB$ is a right angle, $AC = CB$ and $AB = BD$. If $\angle CBD = b$, find the value of b.

$b =$



图一

Figure 1

(iii) 设 x、y 为非零实数。若 x 是 y 的 250%，而 $2y$ 是 x 的 $c\,\%$，求 c 之值。

Let x, y be non-zero real numbers. If x is 250% of y and $2y$ is $c\,\%$ of x, find the value of c.

$c =$

(iv) 若 $\log_p x = 2$， $\log_q x = 3$， $\log_r x = 6$ 及 $\log_{pqr} x = d$， 求 d 之值。

If $\log_p x = 2$, $\log_q x = 3$, $\log_r x = 6$ and $\log_{pqr} x = d$, find the value of d .

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 2 (Group)

香港数学竞赛 (1998 – 99)

决赛项目 2 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 $a=x^4+x^{-4}$ 及 $x^2+x+1=0$，求 a 之值。

If $a=x^4+x^{-4}$ and $x^2+x+1=0$, find the value of a.

$a =$

(ii) 若 $6^b+6^{b+1}=2^b+2^{b+1}+2^{b+2}$，求 b 之值。

If $6^b+6^{b+1}=2^b+2^{b+1}+2^{b+2}$, find the value of b.

$b =$

(iii) 设 c 为质数。若 $11c+1$ 是一正整数之平方，求 c 之值。

Let c be a prime number. If $11c+1$ is the square of a positive integer, find the value of c.

$c =$

(iv) 设 d 为奇质数。若 $89-(d+3)^2$ 是一整数之平方，求 d 之值。

Let d be an odd prime number. If $89-(d+3)^2$ is the square of an integer, find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 3 (Group)

香港数学竞赛 **(1998 – 99)**

决赛项目 **3 (**团体**)**

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 设 a 为 100 以内所有既是平方数亦是立方数的正整数之个数。求 a 之值。

Let a be the number of positive integers less than 100 such that they are both square and cubic numbers . Find the value of a .

$a =$

(ii) 数列 $\{a_k\}$ 之定义如下：

$$a_1 = 1，\ a_2 = 1，\text{而}\ a_k = a_{k-1} + a_{k-2}\ (k > 2)。$$

若 $a_1 + a_2 + \cdots + a_{10} = 11a_b$，求 b 之值。

The sequence $\{a_k\}$ is defined as :

$$a_1 = 1,\ a_2 = 1 \text{ and } a_k = a_{k-1} + a_{k-2}\ (k > 2).$$

If $a_1 + a_2 + \cdots + a_{10} = 11a_b$, find the value of b .

$b =$

(iii) 若 c 是 $\log(\sin x)$ 的最大值，其中 $0 < x < \pi$，求 c 之值。

If c is the maximum value of $\log(\sin x)$, where $0 < x < \pi$, find the value of c .

$c =$

(iv) 已知 $x + y = 18$。若 $\sqrt{x} + \sqrt{y}$ 的最大值为 d，求 d 之值。

Given that $x + y = 18$. If the maximum value of $\sqrt{x} + \sqrt{y}$ is d , find the value of d .

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 4 (Group)

香港数学竞赛 (1998 – 99)

决赛项目 4 (团体)

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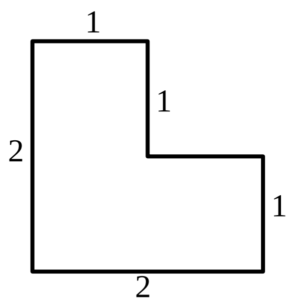
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若以 a 块 L 形的瓷砖 (如图二)，不重迭地拼出一幅与之相似，但面积较大的图形，求 a 的最小的可能数值。

If a tiles of L-shape are used to form a larger similar figure (see Figure 2) without overlapping , find the least possible value of a .

$a =$



图二
Figure 2

(ii) 设 α，β 是 $x^2 + bx - 2 = 0$ 的根，其中 b 为整数。若 $\alpha > 1$ 及 $\beta < -1$，求 b 之值。

Let α , β be the roots of $x^2 + bx - 2 = 0$. If $\alpha > 1$ and $\beta < -1$, find the value of b .

$b =$

(iii) 已知 m，c 是小于 10 的正整数。若 $m = 2c$，且 $0.\dot{m}\dot{c} = \frac{c+4}{m+5}$，求 c 之值。

Given that m , c are positive integers less than 10 . If $m = 2c$ and $0.\dot{m}\dot{c} = \frac{c+4}{m+5}$, find the value of c .

$c =$

(iv) 一个袋子里有 d 个球，其中 x 个是黑球，$x+1$ 是红球，$x+2$ 个是白球。若从袋子里随机抽出一个黑球的概率是小于 $\frac{1}{6}$，求 d 之值。

A bag contains d balls of which x are black, $x+1$ are red and $x+2$ are white. If the probability of drawing a black ball randomly from the bag is less than $\frac{1}{6}$, find the value of d.

$d =$

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Final Event 5 (Group)

香港数学竞赛 (1998 – 99)

决赛项目 5 (团体)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 $x^2 - 2x - P = 0$ 的两根之差为 12，求 P 之值。

If the roots of $x^2 - 2x - P = 0$ differ by 12 , find the value of P .

$P =$

(ii) 已知 $x^2 + ax + 2b = 0$ 和 $x^2 + 2bx + a = 0$ 皆有实根，其中 $a, b > 0$。若 $a + b$ 的最小值为 Q，求 Q 之值。

Given that the roots of $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ are both real and a , $b > 0$. If the minimum value of $a + b$ is Q , find the value of Q .

$Q =$

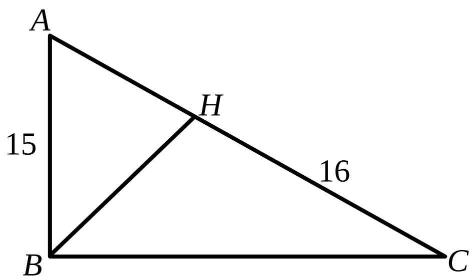
(iii) 若 $R^{2000} < 5^{3000}$，其中 R 为正整数，求 R 的最大值。

If $R^{2000} < 5^{3000}$, where R is a positive integer , find the largest value of R .

$R =$

(iv) 在图三，ΔABC 为一直角三角形，且 $BH \perp AC$ 。若 $AB = 15$，$HC = 16$，且 ΔABC 的面积为 S，求 S 之值。

In Figure 3, ΔABC is a right-angle triangle and $BH \perp AC$. If $AB = 15$, $HC = 16$ and the area of ΔABC is S , find the value of S .



图三
Figure 3

$S =$